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Analytical Representation for Equations of State of Dense Matter

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Abstract. We present an analytical unified representation for 22 equations of state (EoS) of dense matter in neutron stars. Such analytical representations can be useful for modeling neutron star structure in modified theories of gravity with high order derivatives.

1. Introduction

The different possible equations of state (EoS) of dense matter inside neutron stars (NSs) are often given as tabulated data which are fed into the solution of the hydrostatic equilibrium equations via an interpolation technique. While this method works well in the case of general relativity (GR) for which the hydrostatic equilibrium is described by the TOV equations (Tolman 1939; Oppenheimer & Volkoff 1939), it leads to problems in modified theories of gravity, e.g. $f(R)$ theories (de Felice & Tsujikawa 2010) in which the equations describing the hydrostatic equilibrium contain second derivatives of $P(\rho)$ (see e.g. Arapoglu et al. (2011)) where P is the pressure and ρ is the density. In such cases the order of the polynomial used in interpolation technique may effect the results if it is not sufficiently high and the linear interpolation technique will certainly fail. It is then necessary to employ analytical representations of the EoS that are sufficiently many times differentiable. In this work, we provide an accurate unified analytical representation for 22 EoS.

2. Analytical Representation of EoS

A unified analytical representation for two EoS, FPS and SLY, were provided by Haensel & Potekhin (2004) (hereafter HP04) where the authors provide a relation between $\zeta = \log(P/\text{dyn cm}^{-2})$ and $\xi = \log(\rho/\text{g cm}^{-3})$ with 18 free parameters. The unified representation we provide here is an extension of HP04 with 23 parameters 12 of which are fixed to represent low density regimes of BPS (Baym et al. 1971) and NV (Negele & Vautherin 1973). The rest 11 free parameters are used to fit the different EoS at high density regimes and to match them. The function we use to represent EoS for NS is

$$\zeta = \zeta_{\text{low}} f_0(a_1(\xi - c_{11})) + f_0(a_2(c_{12} - \xi)) \zeta_{\text{high}} \quad (1)$$

where $f_0(x) = 1/(1 + \exp x)$ is the matching function (also used in HP04).

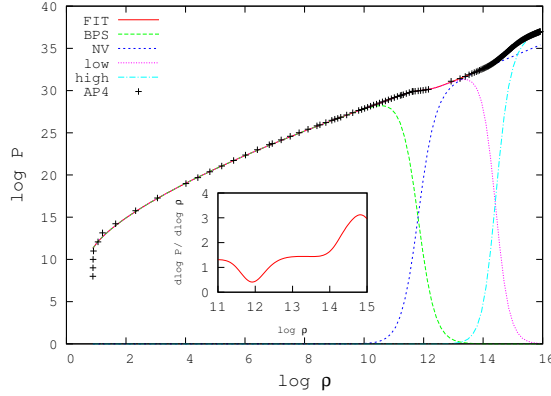


Figure 1. The EoS AP4 (+) and its analytical representation (red line). The purple dotted line and turquoise dash-dotted line describe low and high density regimes, respectively. The small figure shows the adiabatic index $\Gamma = d\zeta/d\xi$ versus ξ .

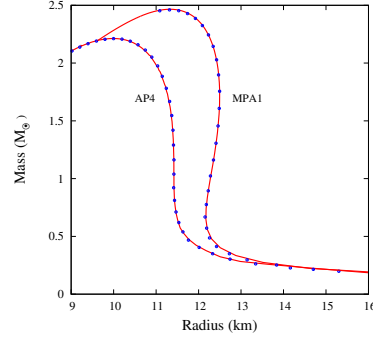


Figure 2. M-R relation for EoS, AP4 and MPA1. The red line is obtained by the analytical representation of the EoSs as presented in this work while the blue circles are obtained by the tabulated EoS.

Here

$$\zeta_{\text{low}} = [c_1 + c_2(\xi - c_3)^{c_4}] f_0(c_5(\xi - c_6)) + (c_7 + c_8\xi) f_0(c_9(c_{10} - \xi)) \quad (2)$$

and

$$\zeta_{\text{high}} = (a_3 + a_4\xi) f_0(a_5(a_6 - \xi)) + (a_7 + a_8\xi + a_9\xi^2) f_0(a_{10}(a_{11} - \xi)) \quad (3)$$

describe the low and high density regimes, respectively. We provide the values of the fit parameters c_i and a_i for $\xi > 5$ in Table 1 and Table 2.

In Figure 1, we show our results for fitting the EoS data of AP4 with Equation (1). In Figure 2, we compare the M-R relations obtained by the analytical representation with that obtained by feeding the EoS data via interpolation technique. The maximum relative error is $\sim 0.05\%$ near the maximum mass.

3. Discussion and Conclusion

The unified analytical expression presented in this work is an accurate representation of many EoS suggested for NSs. The analytical representation is preferable for solving the structure of NSs in modified theories of gravity where hydrostatic equilibrium equations are of 4th order. In such cases the usual interpolation technique will fail because the high order derivatives may not be continuous if the order of the polynomial used for interpolation is not sufficiently high.

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References

- Arapoglu, A. S., Deliduman, C., & Eksi, K. Y. 2011, JCAP, 2011, 020
 Baym, G., Pethick, C., & Sutherland, P. 1971, ApJ, 170, 299
 de Felice, A., & Tsujikawa, S. 2010, Living Reviews in Relativity, 13, 3
 Haensel, P., & Potekhin, A. Y. 2004, A&A, 428, 191
 Negele, J. W., & Vautherin, D. 1973, Nuclear Physics A, 207, 298
 Oppenheimer, J. R., & Volkoff, G. M. 1939, Physical Review, 55, 374
 Tolman, R. C. 1939, Physical Review, 55, 364

Table 1. Parameters of the fit for low regimes

c ₁	10.6557
c ₂	3.7863
c ₃	0.8124
c ₄	0.6823
c ₅	3.5279
c ₆	11.8100
c ₇	12.0584
c ₈	1.4663
c ₉	3.4952
c ₁₀	11.8007
c ₁₁	14.4114
c ₁₂	14.4081

Table 2. Parameters of the fit

EoS	AP1	AP2	AP3	AP4	engvik	gm1nph	gm2nph	gm3nph	mpa1	ms00	ms2
a ₁	4.3290	4.3290	6.3293	4.3290	5.0487	11.4832	10.9801	11.5163	5.2934	16.6491	14.0084
a ₂	4.3622	4.3622	6.3467	4.3622	5.0838	11.4006	10.9233	11.4210	5.3319	16.2520	13.8422
a ₃	138.1760	184.1600	174.2030	9.1131	-100.7550	-100.7660	-100.9130	-100.8670	87.7901	8.0809	16.5970
a ₄	-10.1093	-13.4285	-15.0960	-0.4751	9.2091	11.8432	15.6165	16.0001	-5.8466	-0.5036	-1.0943
a ₅	6.0097	5.8719	5.2453	3.4614	1.5946	1.2081	1.0707	0.9211	2.7232	6.8715	5.6701
a ₆	14.0120	13.8551	13.8962	14.8800	13.9137	12.7088	11.9067	11.7186	15.0804	14.7513	14.8169
a ₇	-411.1380	-372.6920	-377.4200	21.3141	462.5180	462.0230	461.2360	461.2870	428.4130	-39.9099	-56.3794
a ₈	48.0721	40.4186	44.3688	0.1023	-48.4401	-51.9392	-55.7395	-55.2969	-57.6403	8.0314	9.6159
a ₉	-1.1630	-0.8061	-0.8911	0.0495	1.1892	1.2438	1.2502	1.2024	2.0957	-0.2022	-0.2332
a ₁₀	4.7514	4.7456	4.5062	4.9401	5.0896	5.0896	5.0897	5.0897	5.0588	5.0102	-3.8369
a ₁₁	10.7234	10.7503	13.6882	10.2957	10.2277	10.2277	10.2276	10.2276	10.2727	10.3408	23.1860
	ms1506	pal2	pclnphq	wff1	wff2	wff3	wff4	schaf1	schaf2	prakdat	ps
a ₁	15.4744	11.3079	11.3180	2.8144	3.9694	5.2934	5.0395	11.9204	13.3358	12.3641	12.3769
a ₂	15.1705	11.1963	11.2234	2.8539	4.0017	5.2912	5.0512	11.7992	13.1285	12.2125	12.0277
a ₃	2.8727	-0.3393	-9.2769	-6.6340	-6.1548	-5.8771	-5.9160	-11.7736	-6.0249	-9.1898	-4.5181
a ₄	-0.2014	0.0149	0.6194	9.3448	20.5349	29.4964	29.5087	3.1670	2.7698	1.0249	2.6750
a ₅	4.2468	5.0914	7.9411	0.8728	0.5724	0.2267	0.1944	-12.0189	-13.5535	2.5784	-11.4273
a ₆	15.1919	14.5066	15.2677	15.2163	15.3019	17.8721	21.5182	14.6274	14.7235	13.6433	14.5886
a ₇	-83.9695	-68.4757	-194.2320	-379.9680	-377.0480	-380.2820	-381.1690	15.3263	-398.4300	44.5552	-84.9113
a ₈	13.8824	11.8013	28.9879	77.0628	85.8006	65.3388	61.2652	0.5165	55.1217	-2.3817	13.9429
a ₉	-0.3959	-0.3266	-0.9137	-3.5619	-4.5011	-3.1750	-2.6609	0.0521	-1.7487	0.0902	-0.3964
a ₁₀	-4.3087	-4.3089	-4.3181	-4.3181	-4.3181	-4.3181	-4.3181	12.1722	13.6495	4.8729	11.5048
a ₁₁	23.3603	23.3604	23.3626	23.3626	23.3626	23.3626	23.3626	14.6270	14.7214	10.3969	14.5874